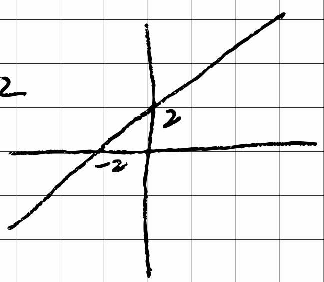
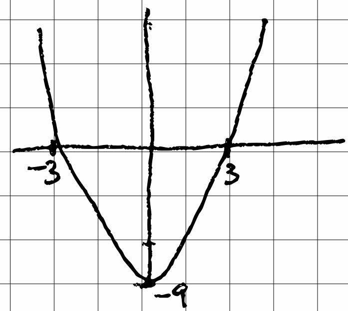
Repassem :

(1) $(x+2)(x^2-9)<0$

si $\begin{cases}(x+2)>0\\(x^2-9)<0\end{cases}$ ó $\begin{cases}(x+2)<0\\(x^2-9)>0\end{cases}$





$x+2>0$ $(-2,+\infty)$

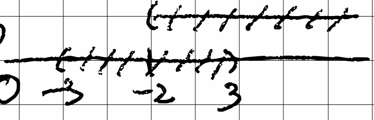
$x+2<0$ $(-\infty,-2)$

$x^2-9>0$ $(-\infty,-3)\cup(3,+\infty)$

$x^2-9<0$ $(-3,3)$

$x+2>0$

$x^2-9<0$

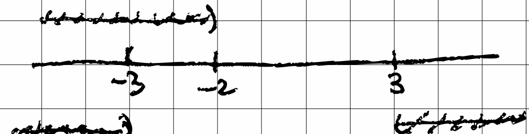


$(-2,3)$

$\cup$

$x+2<0$

$x^2-9>0$



$(-\infty,-3)$

$$\boxed{(-\infty,-3)\cup(-2,3)}$$

② $f(x) = \sqrt{\dfrac{x-2}{x-1}}$

Domini són $x \in \mathbb{R}$ tals que $\dfrac{x-2}{x-1} \geq 0$

és a dir,

$$\begin{cases} x-2 \geq 0 \\ x-1 > 0 \end{cases} \quad \text{ó} \quad \begin{cases} x-2 \leq 0 \\ x-1 < 0 \end{cases}$$

$$\begin{cases} x \geq 2 \\ x < 1 \end{cases} \quad \text{ó} \quad \begin{cases} x \leq 2 \\ x < 1 \end{cases}$$

	$x-2$	$x-1$	$\frac{x-2}{x-1}$	
$(-\infty, 1)$	$-$	$-$	$+$	✓
1	$-$	0		
$(1,2)$	$-$	$+$	$-$	
2	0	$+$	0	✓
$(2,+\infty)$	$+$	$+$	$+$	✓

$$\boxed{(-\infty, 1) \cup [2, +\infty)}$$

③ $P(t)$ = població, a l'any t

$P(0) = P_0$
$P(1) = 1'02 P_0$

$P(t) = (1'02)^t P_0$

$(1'02)^t \cancel{P_0} = 2\cancel{P_0}$

$t = \log_{1'02} 2 = \dfrac{\ln 2}{\ln 1'02} = \boxed{35 \text{ anys}}$

④ $\lim\limits_{x\to\infty} \left(\dfrac{x^2+5}{x^2+1}\right)^{x^2}$ és de tipus 1^∞

$$= e^{\lim\limits_{x\to\infty} \left(\frac{x^2+5}{x^2+1} - 1\right)\cdot x^2} = e^{\lim\limits_{x\to\infty} \frac{4}{x^2+1}\cdot x^2} =$$

$$\left[\frac{x^2+5}{x^2+1} - 1 = \frac{x^2+5-x^2-1}{x^2+1} = \frac{4}{x^2+1}\right]$$

$$= e^{\lim\limits_{x\to\infty} \frac{4x^2}{x^2+1}} = \boxed{e^4}$$

(5) $$f(x) = \begin{cases} |x| & x<1 \\ (x-1)^2+2 & x\geq 1 \end{cases}$$

$$\lim_{x\to 1^+} f(x) = \lim_{\substack{x\to 1^+ \\ x>1}} (x-1)^2+2 = (1-1)^2+2 = 2$$

$$\lim_{x\to 1^-} f(x) = \lim_{\substack{x\to 1 \\ x<1}} |x| = 1$$

$$f(1) = 2$$

$$\left.\begin{array}{l} \lim_{x\to 1^+} f(x) = 2 \\ \lim_{x\to 1^-} f(x) = 1 \end{array}\right\} \neq$$ $\boxed{\text{no es continua en } x=1}$

$$f(2) = (2-1)^2+2 = 3$$

$$\lim_{x\to 2^+} f(x) \underset{2\geq 1}{=} (2-1)^2+2 = 3$$

$$\lim_{x\to 2^-} f(x) \underset{2\geq 1}{=} (2-1)^2+2 = 3$$

es continua en $x=2$

⑥ $f(x) = \dfrac{xe^{-x^2}}{x+4}$ Domini $\mathbb{R} \setminus \{-4\}$
$(-\infty,-4) \cup (-4,+\infty)$

$$\lim_{x\to-\infty} \frac{xe^{-x^2}}{x+4} = \lim_{x\to-\infty} \frac{x}{e^{x^2}(x+4)} = \frac{\infty}{\infty}$$

$$= \lim_{x\to-\infty} \frac{x/e^{x^2}}{x+4} = \frac{0^-}{-\infty} = +\infty$$

e^{x^2} guanya a qualsevol potència

$$\lim_{x\to+\infty} \frac{xe^{-x^2}}{x+4} = \lim_{x\to+\infty} \frac{\frac{x}{e^{x^2}}}{x+4} = \frac{0^+}{+\infty} = +\infty$$

No hi ha assímptotes horitzontals

$$\lim_{\substack{x\to-4^+ \\ x>-4}} \frac{xe^{-x^2}}{x+4} = \frac{-4e^{-16}}{0^+} = -\infty$$

$$\lim_{\substack{x\to-4^- \\ x<-4}} \frac{xe^{-x^2}}{x+4} = \frac{-4e^{-16}}{0^-} = +\infty$$

$\boxed{\text{Assímptota vertical } x=-4}$

Obliqua? $\lim_{x\to\pm\infty} \dfrac{\frac{xe^{-x^2}}{x+4}}{x} = \lim_{x\to\infty} \dfrac{\frac{x}{e^{x^2}}}{x^2+4x} = \dfrac{0}{\infty} = \infty$

No hi ha obliqües

$\boxed{\text{Només té una assímptota}}$

(7) $$\left|\frac{x-120}{4}\right| > 0.25$$

$\frac{x-120}{4} > 0.25$ $\quad$ $x > 4\cdot 0.25 + 120$

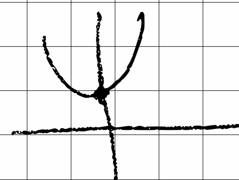
$x > 121$

$\frac{x-120}{4} < -0.25$ $\quad$ $x < -1 + 120$

$x < 119$

menys de 119 o més de 121

(8) $(3x^2+8)(x^2-4x+3) > 0$

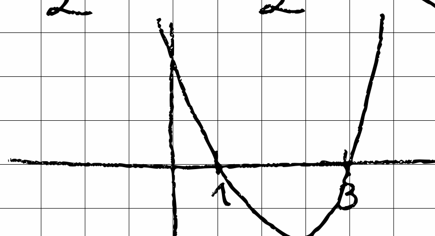


$3x^2+8=0$ no té solució

$3x^2+8 > 0$ sempre.

Alshores només veure quan $x^2-4x+3>0$.

$$x = \frac{4 \pm \sqrt{16-12}}{2} = \frac{4\pm 2}{2} = \begin{cases} 3 \\ 1 \end{cases}$$



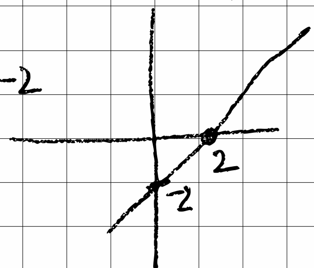
$x^2-4x+3>0$ si $x<1$ o $x>3$

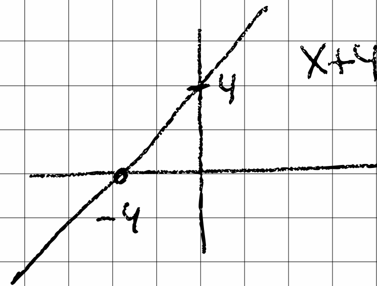
$$\boxed{(-\infty,1)\cup(3,+\infty)}$$

9. $\ln\left(\frac{x-2}{x+4}\right)$ està definit quan

$$\frac{x-2}{x+4} > 0$$

$x-2$





$x+4$

$x-2>0 \quad (2,+\infty)$ $\qquad x+4>0 \quad (-4,+\infty)$

$x-2<0 \quad (-\infty,2)$ $\qquad x+4<0 \quad (-\infty,-4)$

	$x-2$	$x+4$	$\frac{x-2}{x+4}$	
$(-\infty,-4)$	$-$	$-$	$+$	✓
-4	$-$	0		
$(-4,2)$	$-$	$+$	$-$	
2	0	$+$	0	
$(2,+\infty)$	$+$	$+$	$+$	✓

$$\boxed{(-\infty,-4)\cup(2,+\infty)}$$

(10)

$$P(0) = 100$$

$$P(t) = 100 \cdot (1.03)^t$$

$$100 \cdot (1.03)^t = 10.000$$

$$(1.03)^t = 100$$

$$t = \log_{1.03} 100 = \frac{\ln 100}{\ln 1{,}03} = 155'79$$

$$\approx \boxed{156}$$

(11) $\sin 3x \sim 3x$ quan $x \to 0$ ja que aleshores $3x \to 0$

$\sin 4x^2 \sim 4x^2$ ja que $4x^2 \to 0$ per $x \to 0$

$$\lim_{x \to 0} \frac{\sin^4 3x \cos 4x}{x^2 \sin 4x^2} = \lim_{x \to 0} \frac{(3x)^4 \cdot \cos 4x}{x^2 \cdot 4x^2} =$$

$$= \lim_{x \to 0} \frac{81 x^4 \cdot \cos 4x}{4 x^4} = \lim_{x \to 0} \frac{81 \cos 4x}{4} =$$

$$= \frac{81}{4} \cdot 1 = \boxed{\frac{81}{4}}$$

(12) $f(x) = \dfrac{x^2+x+2}{x^2-4x} = \dfrac{x^2+x+2}{x(x-4)}$

$$\lim_{x\to+\infty} \frac{x^2+x+2}{x^2-4x} \underset{\frac{\infty}{\infty}}{=} \lim_{x\to\infty} \frac{1+\frac{1}{x}+\frac{2}{x^2}}{1-\frac{4}{x}} = 1$$

$$\lim_{x\to-\infty} \frac{x^2+x+2}{x^2-4x} = 1$$

$\boxed{\text{Assíptota horitzontal } y=1}$

Domini $(-\infty,0)\cup(0,4)\cup(4,+\infty)$
$\mathbb{R}-\{0,4\}$

$$\lim_{x\to0^-} \frac{x^2+x+2}{x(x-2)} = \frac{2}{0^-\cdot(-2)} = +\infty$$

$$\lim_{x\to0^+} \frac{x^2+x+2}{x(x-2)} = \frac{2}{0^+(-2)} = -\infty$$

$\boxed{\text{Assíptota vertical } x=0}$

$$\lim_{\substack{x\to4^-\\ x<4}} \frac{x^2+x+2}{x(x-4)} = \frac{22}{4\cdot0^-} = -\infty$$

$$\lim_{\substack{x\to4^+\\ x>4}} \frac{x^2+x+2}{x(x-4)} = \frac{22}{4\cdot0^+} = +\infty$$

$\boxed{\text{Assíptota vertical } x=4}$

(13) La funció que descriu el número de passatgers en funció del preu del bitllet és lineal una recta que passa per

$(200, 100)$ i té pendent $\frac{-2}{5}$

$$P(x) = 100 + \left(-\frac{2}{5}\right)(x - 200)$$

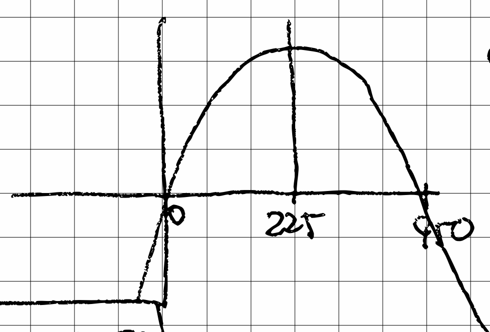
x preu del bitllet

$$\boxed{P(x) = -\frac{2}{5}x + 180}$$

$$\text{Benefici}(x) = \text{preu bitllet} \cdot \text{número passatgers}$$
$$= x \cdot \left(-\frac{2}{5}x + 180\right)$$

Benefici $= 0$ si $x = 0$ (bitllets gratis)

si $x = 450$ (cap passatger)



Gràfica del benefici

$$\boxed{\text{Màxim a } x = 225}$$

(14) $2\ln(y) = \ln(y+1) + x$

$$\ln(y^2) = \ln(y+1) + x$$

$$\ln(y^2) - \ln(y+1) = x$$

$$\ln\left(\frac{y^2}{y+1}\right) = x \iff \frac{y^2}{y+1} = e^x$$

$$y^2 = e^x(y+1), \quad y^2 = e^x y + e^x$$

$$y^2 - e^x y - e^x = 0$$

$$y = \frac{e^x \pm \sqrt{e^{2x} + 4e^x}}{2}$$

Tenim dos possibles valor $y = \dfrac{e^x + \sqrt{e^{2x} + 4e^x}}{2}$

$$y = \frac{e^x - \sqrt{e^{2x} + 4x}}{2}$$

(15) $f(x) = \dfrac{x+1}{4x-5} \qquad g(x) = 3e^{2x-1}$

$$(f\circ g)(x) = f(g(x)) = f(3e^{2x-1}) = \frac{3e^{2x-1}+1}{12e^{2x-1}-5}$$

$$(g\circ f)(x) = g(f(x)) = 3e^{2\cdot\frac{x+1}{4x-5}-1}$$

$$= 3e^{\frac{2x+2-4x+5}{4x-5}} = 3e^{\frac{-2x+7}{4x-5}}$$

Anem a calcular f^{-1}

$$y = \frac{x+1}{4x-5} \; ; \quad 4xy - 5y = x+1$$

$$4xy - x = 1 + 5y$$

$$x(4y-1) = 1 + 5y$$

$$x = \frac{1+5y}{4y-1}$$

$$f^{-1}(x) = \frac{1+5x}{4x-1}$$

Anem a calcular g^{-1}

$$y = 3e^{2x-1} \; ; \quad \frac{y}{3} = e^{2x-1}$$

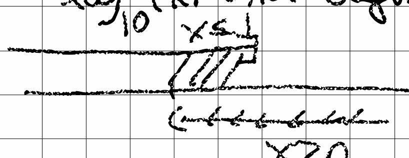
$$2x-1 = \ln\left(\frac{y}{3}\right) \quad \text{si } y>0 \; ✓$$

$$x = \frac{\ln\left(\frac{y}{3}\right)+1}{2}$$

$$g^{-1}(x) = \frac{\ln\left(\frac{x}{3}\right)+1}{2}$$

16 $\sqrt{1-x}$ està definit si $1-x \geq 0$, $x \leq 1$

$\log_{10}(x)$ està definit si $x > 0$



$\boxed{\text{Aleshores les dues condicions alhora } (0,1]}$

(17)

(a) $$\lim_{x\to 0}\left(\frac{\sqrt{3+x}-\sqrt{3}}{x}\right)^3 = \left(\lim_{x\to 0}\frac{\sqrt{3+x}-\sqrt{3}}{x}\right)^3 =$$

$= ???$

$\frac{0}{0}$

Calculem $$\lim_{x\to 0}\frac{\sqrt{3+x}-\sqrt{3}}{x} = \lim_{x\to 0}\frac{(3+x)-3}{x(\sqrt{3+x}+\sqrt{3})} =$$

$$= \lim_{x\to 0}\frac{x}{x(\sqrt{3+x}+\sqrt{3})} = \lim_{x\to 0}\frac{1}{\sqrt{3+x}+\sqrt{3}}$$

$$= \frac{1}{2\sqrt{3}}$$

Aleshores $$\lim_{x\to 0}\left(\frac{\sqrt{3+x}-\sqrt{3}}{x}\right)^3 = \left(\frac{1}{2\sqrt{3}}\right)^3 = \frac{1}{8\cdot 3\sqrt{3}}$$

$$= \frac{\sqrt{3}}{8\cdot 9} = \frac{\sqrt{3}}{72}$$

(b) $\lim_{x\to\infty}\left(\frac{x^3+2x+1}{3+x^3}\right)^{4x} = \underset{1^\infty}{??} =$

$= e^{\lim_{x\to\infty}\left(\frac{x^3+2x+1}{3+x^3}-1\right)4x}$

$\left(\frac{x^3+2x+1}{3+x^3}-1\right)4x = \left(\frac{x^3+2x+1-3-x^3}{3+x^3}\right)4x =$

$= \left(\frac{2x-2}{3+x^3}\right)4x = \frac{8x^2-8x}{x^3+3}$

$= e^{\lim_{x\to\infty}\frac{8x^2-8x}{x^3+3}} = e^{\lim_{x\to+\infty}\frac{8/x-8/x^2}{1+3/x^3}} =$

$= e^0 = \underline{1}$

(c) $\lim_{x\to 0}\frac{\cos^2 x}{x^2} = \frac{1}{0^+} = +\infty$

(d) $\lim_{x\to+\infty}\frac{\sqrt{x^5+x}+x}{\sqrt{2x^3+5x}} = \lim_{x\to+\infty}\frac{\frac{\sqrt{x^5+x}+x}{\sqrt{x^5}}}{\frac{\sqrt{2x^3+5x}}{\sqrt{x^5}}} =$

$= \lim_{x\to\infty}\frac{\sqrt{1+\frac{1}{x^4}}+\frac{1}{\sqrt{x^3}}}{\sqrt{\frac{2}{x^2}+\frac{5}{x^4}}} = \frac{1}{0} = +\infty$

(e) $\lim\limits_{x\to-\infty} \dfrac{2^{x+1}+3^x}{3^x-1} = \dfrac{0+0}{0-1} = \dfrac{0}{-1} = 0$

(f) $\lim\limits_{x\to+\infty} \dfrac{2^{x+1}+3^x}{3^x-1} = \lim\limits_{x\to+\infty} \dfrac{2\cdot 2^x+3^x}{3^x-1} =$

$\dfrac{\infty}{\infty}$

$= \lim\limits_{x\to+\infty} \dfrac{\dfrac{2\cdot 2^x+3^x}{3^x}}{\dfrac{3^x-1}{3^x}} = \lim\limits_{x\to+\infty} \dfrac{2\cdot\left(\frac{2}{3}\right)^x+1}{1-\left(\frac{1}{3}\right)^x} =$

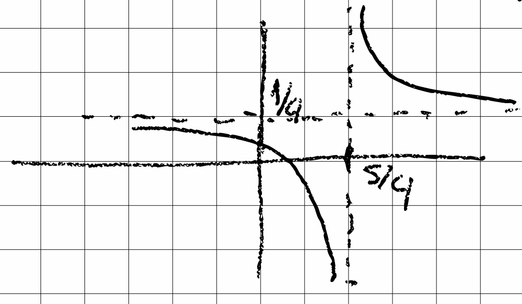
$= \lim\limits_{x\to+\infty} \dfrac{2\cdot 0+1}{1-0} = 1$

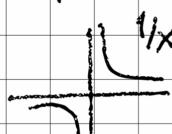
$\lim\limits_{x\to+\infty} a^x = +\infty \quad a>1$

$\lim\limits_{x\to+\infty} a^x = 0 \quad a<1$

(g) $f(x) = \dfrac{x+1}{4x-5} = \dfrac{1}{4} + \dfrac{9/4}{4x-5} = \dfrac{1}{4} + \dfrac{9/4}{4\left(x-\frac{5}{4}\right)}$

$$\begin{array}{r|l} x+1 & 4x-5 \\ \hline -x-5/4 & \frac{1}{4} \\ \hline 9/4 & \end{array}$$



Recordeu 

Anem a calcular asímptotes $4x-5=0$, $x=5/4$

Domini $= \mathbb{R} \setminus \{5/4\} = (-\infty, \frac{5}{4}) \cup (\frac{5}{4}, +\infty)$

$$\lim_{x\to-\infty} \frac{x+1}{4x-5} = \lim_{x\to-\infty} \frac{1+1/x}{4-5/x} = \frac{1}{4}$$

$$\lim_{x\to+\infty} \frac{x+1}{4x-5} = \lim_{x\to+\infty} \frac{1+1/x}{4-5/x} = \frac{1}{4}$$

$$\boxed{\text{Asímptota horitzontal } y=\frac{1}{4}}$$

$$\lim_{x\to\frac{5}{4}^-} \frac{x+1}{4x-5} = \lim_{\substack{x\to\frac{5}{4}^- \\ x<5/4}} \frac{x+1}{4(x-5/4)} = \frac{9/4}{4\cdot 0^-} = -\infty$$

$$\lim_{x\to\frac{5}{4}^+} \frac{x+1}{4x-5} = \lim_{\substack{x\to 5/4^+ \\ x>5/4}} \frac{x+1}{4(x-5/4)} = \frac{9/4}{4\cdot 0^+} = +\infty$$

$$\boxed{\text{Asímptota vertical } x=\frac{5}{4}}$$

19

$$f(x) = \begin{cases} ax^2 + 2x & \text{si } x < 2 \\ \dfrac{2x^2+4}{x+1} & \text{si } x \geq 2 \end{cases}$$

- ax^2+2x és contínua per tot $x \in \mathbb{R}$, en particular si $x<2$

- $\dfrac{2x^2+4}{x+1}$ és contínua per tot $x \in \mathbb{R} \setminus \{-1\}$
$x \in (-\infty,-1) \cup (-1,+\infty)$

però si només mirem $x \geq 2$, $[2,+\infty)$ aleshores

$\dfrac{2x^2+4}{x+1}$ és contínua si $x \geq 2$.

Què passa si $x=2$ per $f(x)$?

$$f(2) = \frac{2\cdot 2^2+4}{2+1} = \frac{12}{3} = 4$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \frac{2x^2+4}{x+1} = 4$$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} ax^2+2x = 4a+4$$

f és contínua en $x=2$ sii $4a+4=4$
és a dir $a=0$.

Si $a=0$, f és contínua per tot $x \in \mathbb{R}$
Si $a \neq 0$, f és contínua excepte en $x=2$. (discontinuïtat de salt)

A LA DRETA : $x \to +\infty$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{2x^2+4}{x+1} = +\infty$$

Mirem si hi ha obliqües :

$$\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{\frac{2x^2+4}{x+1}}{x} = \lim_{x\to+\infty} \frac{2x^2+4}{x^2+x} =$$

$$= \lim_{x\to+\infty} \frac{2+4/x^2}{1+1/x} = 2$$

$m=2$

$$\lim_{x\to+\infty} f(x) - mx = \lim_{x\to+\infty} \frac{2x^2+4}{x+1} - 2x =$$

$$= \lim_{x\to+\infty} \frac{2x^2+4-2x^2-2x}{x+1} = \lim_{x\to+\infty} \frac{-2x+4}{x+1} = -2$$

Assímptota obliqua a dreta $y = 2x-2$

A L'ESQUERRA $x \to -\infty$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} ax^2+2x = \begin{cases} +\infty & a>0 \\ -\infty & a=0 \\ -\infty & a<0 \end{cases}$$

Mirem si hi ha obliqües

$$\lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{ax^2+2x}{x} = \lim_{x\to-\infty} ax+2$$

$$= \begin{cases} +\infty & a<0 \\ 2 & a=0 \\ -\infty & a>0 \end{cases}$$

Si $a=0$, $f(x)=2x$, per tant és asímptota obliqua

$$y=2x$$

Si $a=0$, asímptota obliqua a esquerra $y = 2x$

Si $a \neq 0$ no n'hi ha

Fixeu-vos que $\boxed{f(0)=0}$

- Si $x<0$ $f(x) = x(ax+2)$ aleshores

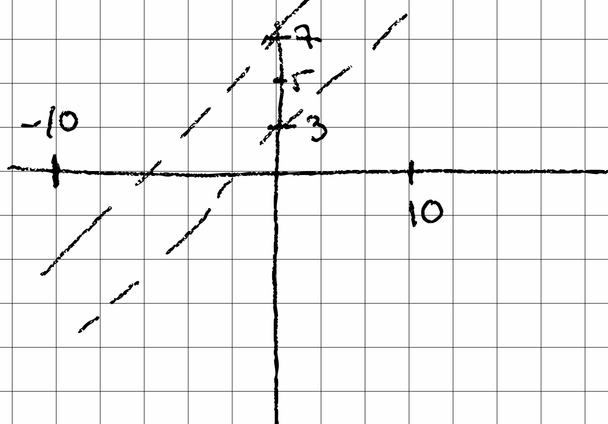
Si $x = \frac{-2}{a} < +2$

$$f\left(-\frac{2}{a}\right)=0$$

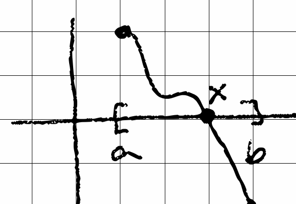
20 Fixeu-vos que

$$-1 \le \cos x \le 2$$
$$-2 \le 2\cos x \le 2$$

$$3x+3 \le 3x+5+2\cos x \le 3x+7$$



Bolzano diu: si f és una contínua en $[a,b]$ i $f(a)$ i $f(b)$ tenen signe contrari aleshores existeix $x \in [a,b]$ amb $f(x)=0$



La funció $f(x) = 3x+5+2\cos x$ és contínua a tot $\mathbb{R}$. Hem de trobar a i b cumplint les condicions de Bolzano.

$$f(-10) = -30+5+2\cos(-10) = -25+2\cos(10) < 0$$
$$f(10) = 30+5+2\cos(10) = 35+2\cos(10) > 0$$

Per tant hi ha $x \in [-10,10]$ tal que $f(x)=0$.